## Definition

**Differential calculus** is the study of **rates of change** of functions, using the tools of **limits** and **derivatives**.

**Applications of differential calculus**

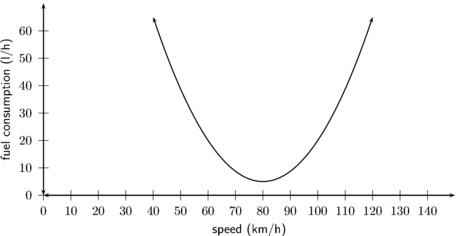
1. **Optimisation problems**

We have seen that differential calculus can be used to determine the stationary points of functions, in order to sketch their graphs. Calculating stationary points also lends itself to the solving of problems that require some variable to be maximised or minimised. These are referred to as optimisation problems.

The fuel used by a car is defined by f(v)=380v2−6v+245f(v)=380v2−6v+245, where vv is the travelling speed in km/h.

What is the most economical speed of the car? In other words, determine the speed of the car which uses the least amount of fuel.

If we draw the graph of this function we find that the graph has a minimum. The speed at the minimum would then give the most economical speed.



We have seen that the coordinates of the turning point can be calculated by differentiating the function and finding the xx-coordinate (speed in the case of the example) for which the derivative is 0.

f′(v)=340v−6f′(v)=340v−6

If we set f′(v)=0f′(v)=0 we can calculate the speed that corresponds to the turning point:

f′(v)0v=340v−6=340v−6=6×403=80f′(v)=340v−60=340v−6v=6×403=80

This means that the most economical speed is 80 km/h80 km/h.

**Finding the optimum point:**

Let f′(x)=0f′(x)=0 and solve for xx to find the optimum point.

To check whether the optimum point at x=ax=a is a local minimum or a local maximum, we find f′′(x)f″(x):

* If f′′(a)<0f″(a)<0, then the point is a local maximum.
* If f′′(a)>0f″(a)>0, then the point is a local minimum.

## ****2. Rates of change****

It is very useful to determine how fast (the rate at which) things are changing. Mathematically we can represent change in different ways. For example we can use algebraic formulae or graphs.

We have learnt how to determine the average gradient of a curve and how to determine the gradient of a curve at a given point. These concepts are also referred to as the average rate of change and the instantaneous rate of change.

Average rate of change =f(x+h)−f(x)(x+h)−xAverage rate of change =f(x+h)−f(x)(x+h)−x

Instantaneous rate of change =limh→0f(x+h)−f(x)hInstantaneous rate of change =limh→0f(x+h)−f(x)h

When we mention rate of change, the instantaneous rate of change (the derivative) is implied. When average rate of change is required, it will be specifically referred to as average rate of change.

### Identity 2

Average velocity Instantaneous velocity =Average rate of change =Instantaneous rate of change =DerivativeAverage velocity =Average rate of change Instantaneous velocity =Instantaneous rate of change =Derivative

Velocity refers to the change in distance (ss) for a corresponding change in time (tt).

v(t)=dsdt=s′(t)v(t)=dsdt=s′(t)

Acceleration is the change in velocity for a corresponding change in time. Therefore, acceleration is the derivative of velocity

a(t)=v′(t)a(t)=v′(t)

This implies that acceleration is the second derivative of the distance.

a(t)=s′′(t)

### [Rectilinear motion](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/rectilinear-motion-diff-calc-dc/v/when-is-a-particle-speeding-up)

Solve problems about motion along a line using the power of differential calculus. For example, given the position of a particle as a function of time s(t), find the particle's maximum velocity.

### [Planar motion](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/planar-motion-diff-calc-dc/v/planar-motion-example-acceleration-vector)

Solve problems about motion on a 2-dimensional plane using the power of differential calculus. For example, given the (x,y) position of a particle as a function of time (x(t),y(t)), find the particle's position when its acceleration is 0.

# [Applications of Calculus](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)

[With calculus, we have the ability to find the effects of changing conditions on a system. By studying these, you can learn how to control a system to make it do what you want it to do. Because of the ability to model and control systems, calculus gives us extraordinary power over the material world.](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)

[Calculus is the language of engineers, scientists, and economists. The work of these professionals has a huge impact on our daily life - from your microwaves, cell phones, TV, and car to medicine, economy, and national defense](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)

**[Biologists](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)**[use differential calculus to determine the exact rate of growth in a bacterial culture when different variables such as temperature and food source are changed. This research can help increase the rate of growth of necessary bacteria, or decrease the rate of growth for harmful and potentially threatening bacteria.](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)

[An](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)**[electrical engineer](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)**[uses integration to determine the exact length of power cable needed to connect two substations that are miles apart. Because the cable is hung from poles, it is constantly curving. Calculus allows a precise figure to be determined.](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)

[An](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)**[architect](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)**[will use integration to determine the amount of materials necessary to construct a curved dome over a new sports arena, as well as calculate the weight of that dome and determine the type of support structure required.](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)

[An](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)**[operations research analyst](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)**[will use calculus when observing different processes at a manufacturing corporation. By considering the value of different variables, they can help a company improve operating efficiency, increase production, and raise profits.](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)

[Obviously, a wide variety of careers regularly use calculus. Universities, the military, government agencies, airlines, entertainment studios, software companies, and construction companies are only a few employers who seek individuals with a solid knowledge of calculus. Even doctors and lawyers use calculus to help build the discipline necessary for solving complex problems, such as diagnosing patients or planning a prosecution case. Despite its mystique as a more complex branch of mathematics, calculus touches our lives each day, in ways too numerous to calculate.](https://www.khanacademy.org/math/differential-calculus/derivative-applications-dc/lhopitals-rule-dc/v/introduction-to-l-hopital-s-rule)